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by

John F. Seely*

*LASL Consultant. Department of Physics, University of Southern California, Los Angeles, CA 90007.

los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87545

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ABSTRACT

A model for the He-Cs charge exchange laser has been formulated in which the coupled photon and particle rate equations are solved numerically for the laser intensities at 584 Å and 2 μ. The photon rate equations include spontaneous and stimulated emission and photoabsorption processes. The particle rate equations include charge exchange, electron impact excitation and ionization, spontaneous and stimulated emission, photoabsorption, and all appropriate back reactions. Input parameters include initial particle densities and temperatures, shapes of the leading edges of the helium plasma and target, relative drift velocity of the plasma and target, and laser length. The laser intensities are calculated as functions of time, depth into the target, and position along the laser axis.

I. INTRODUCTION

In designing experiments for the proposed He-Cs charge exchange laser,[1] it is useful to have a rate equation model of the system in order to establish the practicability of the laser scheme and to fix operating parameters such as initial particle densities and temperatures. Such a theoretical model may also be used to interpret experimental data.

An analytical model[2] based on approximate particle rate equations indicates that population inversions of a few percent and linear gains higher than 1 cm^{-1} are obtainable. In this model, the four rate equations for cesium neutrals, helium ions, helium 1 $^1S_0$ state, and helium 2 $^1P_1$ state are written. After comparing competing reaction rates, only terms due to the fastest collisional reactions are saved in each rate equation. Spontaneous decay is included, but not stimulated emission. The approximate rate equations are then solved analytically for the population inversion and linear gain for the helium 584-Å line. Although this simple analytical model gives useful physical insight into which collisional reactions are most important in determining the population inversion, a more sophisticated numerical calculation is needed in which the coupled photon and particle rate equations, including all appropriate radiative and collisional terms, are solved simultaneously.

II. SCOPE OF THE NUMERICAL MODEL

Of primary interest is the laser intensity as a function of time and position, and the dependence of the laser output on the operating parameters of the system. The collisional and radiative reactions contributing to the rate equations are modeled in detail. Plasma instabilities, magnetohydrodynamic plasma motion, spontaneously generated magnetic fields, and related phenomena are not taken into account in this initial calculation, but may be added in subsequent models.

We anticipate that the plasma electrons will play an important role in ionizing the cesium target atoms, a process that competes with the charge exchange reaction which fills the upper laser state.

One may minimize this effect by increasing the target density to values much greater than the plasma density, so that only a small fraction of the cesium atoms is ionized.[7]

We assume that the cesium target is at rest, and that the helium plasma drifts toward the target with velocity v_o. The plasma ion and electron velocity distributions are assumed to be Maxwellian with constant temperatures T_i and T_e, respectively. Since the cesium temperature is much less than the drift energy ($mv_o^2/2 \sim 1$ keV) and the plasma temperatures ($T_i \sim T_e \sim 50$ eV), we set the cesium temperature equal to zero.

As shown in Fig. 1, we calculate particle densities and laser intensities as functions of x, the penetration depth into the target, and z, the position along the laser axis. Stimulated photons are assumed to be emitted in the + z direction.

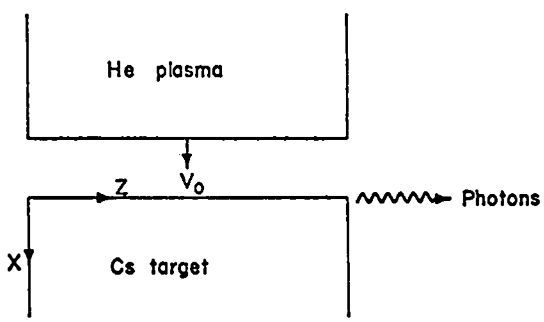


Fig. 1

We treat cesium as a two-level atom composed of the ground state and the 6 $^2P_{1/2,\ 3/2}$ excited state. Helium is treated as a three-level atom composed of the ground state and the 2 ^1S_o and 2 $^1P_1$ excited states. We are primarily interested in the laser transitions 2 $^1P_1 \rightarrow 1\ ^1S_o$ at 584 Å and 2 $^1P_1 \rightarrow 2\ ^1S_o$ at 2 μ.

III. RATE COEFFICIENTS

Consider a cesium atom drifting with velocity $\vec{V}_o$ through a helium plasma. The rate coefficient for the transfer of an electron from the cesium state α to the helium state β is[3]

$$_\alpha R_\beta = \int f_\alpha(\vec{w}) f_\beta(\vec{v}) |\vec{v}-\vec{w}|\ _\alpha Q_\beta(|\vec{v}-\vec{w}|)\ d^3w\ d^3v, \qquad (1)$$

where $_\alpha Q_\beta$ is the charge exchange cross section, $\vec{w}$ is the velocity of the cesium atom, and $\vec{v}$ is the velocity of the helium ions. Since the cesium atom has velocity $\vec{v}_o$, the cesium velocity distribution is a delta function,

$$f_\alpha(\vec{w}) = \delta(\vec{w} - \vec{v}_o). \qquad (2)$$

Substituting Eq. (2) into Eq. (1) and using the delta function to integrate over $\vec{w}$, the rate coefficient becomes

$$_\alpha R_\beta = \int f_\beta(\vec{v}) |\vec{v}-\vec{v}_o|\ _\alpha Q_\beta(|\vec{v}-\vec{v}_o|)\ d^3v. \qquad (3)$$

The drift energy of the cesium atom ($mv_o^2/2 \sim 1$ keV) is much greater than the helium ion temperature ($T_i \sim 50$ eV). We therefore set $|\vec{v} - \vec{v}_o| \cong v_o$ in Eq. (3), and write

$$_\alpha R_\beta = \int f_\beta(\vec{v})\ v_o\ _\alpha Q_\beta(v_o)\ d^3v. \qquad (4)$$

The helium ion distribution is assumed to be Maxwellian with constant temperature T_i,

$$f_\beta(\vec{v}) = (m_i/2\pi T_i)^{3/2} \exp(-m_i v^2/2T_i). \qquad (5)$$

We assume that the charge exchange cross section as a function of energy E may be fitted to a curve of the form

$$_\alpha Q_\beta(E) = U(E-E_c)\ Q_o\ e^{-E/E_Q} \qquad (6)$$

as shown in Fig. 2. Here U(x) is the unit step function, E_c is a threshold energy below which the cross section is zero, and E_Q is the energy at which the cross section has decreased by a factor of e^{-1}. Substituting Eqs. (5) and (6) into Eq. (4) and integrating over v, the rate coefficient for charge exchange becomes

$$_\alpha R_\beta = V_o Q_o\ e^{-E_o/E_Q} U(E_o-E_c),\ E_o = m_i V_o^2/2. \qquad (7)$$

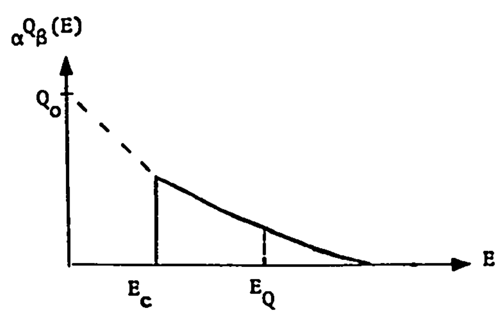


Fig. 2

The cross-section parameters Q_o, E_Q, and E_c entering into the rate coefficient (7) are listed in Table I. The numbering system for the atomic states α and β is given by the second column in Table III. For example, the cross section for the transfer of an electron from the ground state $6\ ^2S_{1/2}$ of cesium (α = 1) to the $2\ ^1P$ state of helium (β = 6) is

$$_1Q_6(E) = U(E-50\ \text{eV})(2\times10^{-15}\ \text{cm}^2)\exp[-E/10^4\ \text{eV}].$$

The charge transfer cross-section parameters listed in Table I are taken from a theoretical calculation[4] and should be viewed as approximate.

TABLE I

CHARGE EXCHANGE CROSS-SECTION PARAMETERS FROM REF. 4

α	β	Q_o, cm^2	E_Q, eV	E_c, eV	Experimental, E, or Theoretical, T
1	4	10^{-17}	10^4	50	T
1	5	2×10^{-15}	10^4	50	T
1	6	2×10^{-15}	10^4	50	T
2	4	10^{-17}	10^4	50	T
2	5	10^{-17}	10^4	50	T
2	6	10^{-17}	10^4	50	T

The rate coefficient for electron impact ionization or excitation from the cesium state α to the cesium state β is

$$_\alpha S^\beta = \int f_\alpha(\vec{w}) f_e(\vec{v}) |\vec{v}-\vec{w}|\, _\alpha Q^\beta(|\vec{v}-\vec{w}|)\, d^3w\, d^3v, \qquad (8)$$

where $f_\alpha(\vec{w})$ is the cesium velocity distribution and $f_e(\vec{v})$ is the plasma electron distribution. Substituting Eq. (2) into Eq. (8),

$$_\alpha S^\beta = \int f_e(\vec{v}) |\vec{v}-\vec{v}_o|\, _\alpha Q^\beta(|\vec{v}-\vec{v}_o|)\, d^3v. \qquad (9)$$

We assume that the electron distribution is Maxwellian with constant temperature T_e, where $T_e \sim 50$ eV. The average velocity of the electron distribution,

$$V_{av} = (8\ T_e/\pi m_e)^{1/2}, \qquad (10)$$

is much greater than the drift velocity. We therefore set $|\vec{v} - \vec{v}_o| \cong v$ in Eq. (9). Again assuming that the cross section may be fitted to the curve given by Eq. (6), the rate coefficient for the electron impact ionization or excitation of cesium from state α to state β is

$$_\alpha S^\beta = v_{av}\ Q_o(1+E_c/T_e+E_c/E_Q)$$
$$\times \exp[-E_c/T_e-E_c/E_Q]/(1+T_e/E_Q)^2. \qquad (11)$$

The rate coefficient for the electron impact ionization or excitation of helium from state α to state β is

$$_\alpha S^\beta = \int f_\alpha(\vec{w}) f_e(\vec{v}) |\vec{v}-\vec{w}|\, _\alpha Q^\beta(|\vec{v}-\vec{w}|)\, d^3v\, d^3w, \qquad (12)$$

where $f_\alpha(\vec{w})$ is the helium velocity distribution with temperature T_i and $f_e(\vec{v})$ is the electron distribution with temperature T_e. If $T_i \le T_e$, then the average helium velocity is much less than the average electron velocity, and the helium atoms appear to the electrons to be nearly stationary. We therefore set $f_\alpha(\vec{w}) = \delta(\vec{w})$ in Eq. (12) and obtain

$$_\alpha S^\beta = \int f_e(\vec{v})\ v\ _\alpha Q^\beta(v)\ d^3v. \qquad (13)$$

Assuming a Maxwellian distribution for the electrons and using Eq. (6), the rate coefficient for the electron impact ionization or excitation of helium is the same form as Eq. (11), the result for cesium. This is because in both derivations, we have effectively assumed that the electron velocity is much greater than the ion velocity.

The cross-section parameters for electron ionization and excitation of cesium and helium are listed in Table II. Cross sections for ionization or excitation from the ground states (α = 1 or 4) are experimental measurements.[3] Other values are estimated using the Darwin model[3] and should be viewed as approximate.

TABLE II

ELECTRON IMPACT EXCITATION AND IONIZATION CROSS-SECTION PARAMETERS FROM REF. 3

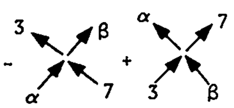
α	β	Q_o, cm^2	E_Q, eV	E_c, eV	Experimental, E, or Theoretical, T
1	3	1×10^{-15}	400	3.9	E
2	3	8×10^{-15}	144	1.4	T
1	4	1×10^{-14}	130	1.4	E
4	7	5×10^{-17}	500	24.6	E
5	7	9.7×10^{-16}	80	4.0	T
6	7	1.3×10^{-15}	70	3.4	T
4	5	1.3×10^{-17}	580	20.6	T
4	6	1.2×10^{-17}	600	21.2	E
5	6	1.4×10^{-14}	0.6	17	T

IV. RATE EQUATION TERMS

In this section, we develop the terms contributing to the particle rate equations due to charge exchange, electron impact ionization and excitation, and the corresponding back reactions.

The change in the number of cesium atoms of type α due to the transfer of an electron to the helium state β may be written[7]

$$\left.\frac{dn_\alpha}{dt}\right|_{cx}^{\beta} = [\; - \;\; + \;\;]$$



$$= [-n_\alpha n_7 \, {}_\alpha R_\beta + n_3 n_\beta \, {}_\beta R_\alpha]$$

$$= -n_\alpha n_7 \, {}_\alpha R_\beta \, [1 - (n_3 n_\beta / n_\alpha n_7)({}_\beta R_\alpha / {}_\alpha R_\beta)] . \quad (14)$$

Here n_α is the number density of particles in state α . Using detailed balancing,[3] the ratio of the forward and backward rate coefficients for charge exchange may be written

$$({}_\alpha R_\beta / {}_\beta R_\alpha) = (g_\beta / g_\alpha) \exp(-E_{\beta-\alpha} / T_i) , \quad (15)$$

where g_α is the degeneracy of state α , E_α is the energy level of state α , and

$$E_{\beta-\alpha} \quad |E_\beta - E_\alpha| .$$

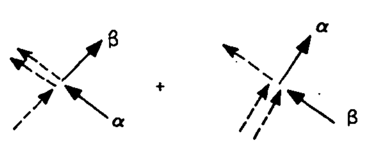
Values for g_α and E_α are given in Table IV. Substituting Eq. (15) into Eq. (14), the change in the number of cesium atoms in state α due to charge exchange with helium ions is

$$\left.\frac{dn_\alpha}{dt}\right|_{cx}^{\beta} = -n_\alpha n_7 \, {}_\alpha R_\beta [1-(n_3 n_\beta / n_\alpha n_7)(g_\alpha / g_\beta) \exp(E_{\beta-\alpha} / T_i)] \quad (16)$$

The charge exchange rate coefficient ${}_\alpha R_\beta$ may be calculated using Eq. (7) and the cross-section parameters in Table I.

The change in number of atoms in state due to electron impact ionization and three-body recombination is

$$\left.\frac{dn_\alpha}{dt}\right|_{ion}^{\beta} = [- \;\; + \;\;]$$



$$= -n_\alpha n_8 \, {}_\alpha S^\beta [1-(n_8 n_\beta / n_\alpha)({}^\beta S_\alpha / {}_\alpha S^\beta)] , \quad (17)$$

where n_8 is the electron density and β identifies the ion (β = 3 for cesium; β = 7 for helium). Using detailed balancing, the ratio of the forward to backward rate coefficients may be written[3]

$$({}_\alpha S^\beta / {}^\beta S_\alpha) = (2\pi m_e T_e / h^2)^{3/2} \, (2g_o / g_\alpha) \exp(-E_{\alpha-\beta} / T_e) , \quad (18)$$

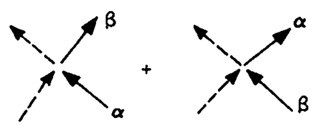
where g_o is the degeneracy of the ground state (g_o = g_1 = 2 for cesium; g_o = g_4 = 1 for helium). Substituting Eq. (18) into (17), the change in number of atoms in state α due to electron impact ionization is

$$\left.\frac{dn_\alpha}{dt}\right|_{ion}^{\beta} = -n_\alpha n_8 \, {}_\alpha S^\beta [1-(n_8 n_\beta / n_\alpha)(h^2 / 2\pi m_e T_e)^{3/2}$$

$$\times \, (g_\alpha / 2g_o) \, \exp(E_{\alpha-\beta} / T_e)] . \quad (19)$$

The change in number of atoms in state α due to electron impact excitation to state β is

$$\left.\frac{dn_\alpha}{dt}\right|_{ex}^{\beta} = [\; - \;\; + \;\;]$$



$$= -n_8 n_\alpha \, {}_\alpha S^\beta [1-(n_\beta / n_\alpha)(g_\alpha / g_\beta) \, \exp(E_{\alpha-\beta} / T_e)] , \quad (20)$$

where we used detailed balancing to write[3]

$$({}_\alpha S^\beta / {}^\beta S_\alpha) = (g_\beta / g_\alpha) \, \exp(-E_{\alpha-\beta} / T_e) . \quad (21)$$

V. PARTICLE RATE EQUATIONS

The terms included in the target rate equations are shown in Table III. These terms are:

(1) Charge exchange between the cesium 6 $^2S_{1/2}$ and 6 ^2P states and the helium 1 ^1S_o, 2 ^1S_o, and 2 $^1P_1$ states,

(2) Electron impact ionization of the cesium 6 $^2S_{1/2}$ and 6 ^2P states,

(3) Electron impact excitation 6 $^2S_{1/2}$ → 6 ^2P,

(4) Spontaneous emission 6 ^2P → 6 $^2S_{1/2}$,

(5) Photoabsorption 6 $^2S_{1/2}$ → ion of 584 Å photon,

and all appropriate back reactions. The rate equations for the cesium ground state is

TABLE III

A CHECK INDICATES THAT THE TERM IS INCLUDED IN THE PARTICLE RATE EQUATION

Particle Species		α	Charge Exchange	Impact Ionization	Impact Excitation	Spontaneous Emission	Stimulated Emission	Photoabsorption
Cesium	$6\ ^2S_{1/2}$	1	✓	✓	✓	✓		✓
	$6\ P$	2	✓	✓	✓	✓		
	Ion	3	✓	✓				✓
Helium	$1\ ^1S_o$	4	✓	✓	✓	✓	✓	
	$2\ ^1S_o$	5	✓	✓	✓	✓	✓	
	$2\ ^1P_1$	6	✓	✓	✓	✓	✓	
	Ion	7	✓	✓				
Photons	584 Å					✓	✓	✓
	2 μ					✓	✓	

$$\frac{\partial n_1}{\partial t} = \sum_{\beta=4,5,6} \left.\frac{dn_1}{dt}\right|_{cx}^{\beta} + \left.\frac{dn_1}{dt}\right|_{ion}^{3} + \left.\frac{dn_1}{dt}\right|_{ex}^{2} + n_2\ {}^2A_1 - 2k_1\ I_1/\hbar\nu_1 , \qquad (22)$$

where the fourth term is due to spontaneous decay of the $6\ ^2P$ ($\alpha = 2$) state, and ${}^2A_1$ is the transition probability given in Table V. The last term, due to photoabsorption, is derived in Section VI. The rate equation for the $6\ ^2P$ state of cesium is

$$\frac{\partial n_2}{\partial t} = \sum_{\beta=4,5,6} \left.\frac{dn_2}{dt}\right|_{cx}^{\beta} + \left.\frac{dn_2}{dt}\right|_{ion}^{3} - \left.\frac{dn_1}{dt}\right|_{ex}^{2} - n_2\ {}^2A_1 . \qquad (23)$$

Adding Eqs. (22) and (23), the rate equation for cesium ions is

$$\frac{\partial n_3}{\partial t} = -\sum_{\substack{\alpha=1,2\\ \beta=4,5,6}} \left.\frac{dn_\alpha}{dt}\right|_{cx}^{\beta} - \sum_{\alpha=1,2} \left.\frac{dn_\alpha}{dt}\right|_{ion}^{3} + 2k_1\ I_1/\hbar\nu_1 . \qquad (24)$$

In writing the cesium rate equations, we have assumed that the cesium atoms are at rest.

The terms included in the plasma rate equations are shown in Table III. These terms are:

(1) Charge exchange between the helium $1\ ^1S_o$, $2\ ^1S_o$, and $2\ ^1P_1$ states and the cesium $6\ ^2S_{1/2}$ and $6\ ^2P$ states,

(2) Electron impact ionization of the helium $1\ ^1S_o$, $2\ ^1S_o$, and $2\ ^1P_1$ states,

(3) Electron impact excitations $1\ ^1S_o \rightarrow 2\ ^1S_o$, $1\ ^1S_o \rightarrow 2\ ^1P_1$, and $2\ ^1S_o \rightarrow 2\ ^1P_1$,

(4) Spontaneous emission $2\ ^1P_1 \rightarrow 1\ ^1S_o$ and $2\ ^1P_1 \rightarrow 2\ ^1S_o$,

(5) Stimulated transitions between the $2\ ^1P_1$ and $1\ ^1S_o$ states, and between the $2\ ^1P_1$ and $2\ ^1S_o$ states, and all appropriate back reactions. The rate equation for the helium ground state is

$$\frac{\partial n_4}{\partial t} + v_o \frac{\partial n_4}{\partial x} = -\sum_{\alpha=1,2} \left.\frac{dn_\alpha}{dt}\right|_{cx}^{4} + \left.\frac{dn_4}{dt}\right|_{ion}^{7} + \sum_{\beta=5,6} \left.\frac{dn_4}{dt}\right|_{ex}^{\beta} + n_6\ {}^6A_4 + g_1 I_1/\hbar\nu_1 , \qquad (25)$$

where the last term is due to stimulated emission from the $2\ ^1P_1$ state to the $1\ ^1S_o$ state at 584 Å. This term is determined as a function of the linear gain g_1 and the laser intensity I_1 in Section VI. The rate equation for the helium $2\ ^1S_o$ is

$$\frac{\partial n_5}{\partial t} + v_o \frac{\partial n_5}{\partial x} = -\sum_{\alpha=1,2} \left.\frac{dn_\alpha}{dt}\right|_{cx}^{5} + \left.\frac{dn_5}{dt}\right|_{ex}^{6} - \left.\frac{dn_4}{dt}\right|_{ex}^{5} + \left.\frac{dn_5}{dt}\right|_{ion}^{7}$$

$$+ n_6 \, {}^6A_5 + g_2 \, I_2/\hbar\nu_2 . \qquad (26)$$

The rate equation for the helium 2 $^1P_1$ state is

$$\frac{\partial n_6}{\partial t} + v_o \frac{\partial n_6}{\partial x} = - \sum_{\alpha=1,2} \left.\frac{dn_\alpha}{dt}\right|_{cx}^{6} + \left.\frac{dn_6}{dt}\right|_{ion}^{7} - \sum_{\beta=4,5} \left.\frac{dn_\beta}{dt}\right|_{ex}^{6}$$

$$- n_6({}^6A_4 + {}^6A_5) - \sum_{i=1,2} g_i I_i/\hbar\nu_i . \qquad (27)$$

Adding Eqs. (25), (26), and (27) the rate equation for helium ions is

$$\frac{\partial n_7}{\partial t} + v_o \frac{\partial n_7}{\partial x} = \sum_{\substack{\alpha=1,2\\ \beta=4,5,6}} \left.\frac{dn_\alpha}{dt}\right|_{cx}^{\beta} - \sum_{\beta=4,5,6} \left.\frac{dn_\beta}{dt}\right|_{ion}^{7} . \qquad (28)$$

Regarding the free electrons, we are primarily concerned with those electrons with sufficient energy to cause excitation or ionization events which compete with the charge exchange and laser transitions of interest. In writing the rate equation for the hot electrons, we assume that a hot electron loses an average of about 10 eV during each excitation or ionization event, and thus participates in (T_e/10eV) such events:[7]

$$\frac{\partial n_8}{\partial t} + v_o \frac{\partial n_8}{\partial x} = \frac{10eV}{T_e} \left[\sum_{\alpha=1,2} \left.\frac{dn_\alpha}{dt}\right|_{ion}^{3} + \sum_{\beta=4,5,6} \left.\frac{dn_\beta}{dt}\right|_{ion}^{7} + \left.\frac{dn_1}{dt}\right|_{ex}^{2} + \left.\frac{dn_4}{dt}\right|_{ex}^{5} + \sum_{\alpha=4,5} \left.\frac{dn_\alpha}{dt}\right|_{ex}^{6} \right] . \qquad (29)$$

VI. PHOTON RATE EQUATIONS

In this section, we establish the rate equations for the laser intensities I_1 and I_2 of the two helium lines with wavelengths λ_1 = 584 Å and λ_2 = 2 μ, as shown in Table V. According to Ref. 5, the equation for the laser field strength E_i^2 is

$$\frac{\partial E_i^2}{\partial t} + c \frac{\partial E_i^2}{\partial z} = c \, g_i \, E_i^2 - 2 \, ck_i E_i^2 , \qquad (30)$$

where k_i is the absorption coefficient and the linear gain is

$$g_i = (3/2)(\ln 2/\pi)^{1/2} (\lambda_i^2 \, {}^\alpha A_\beta n_o/\Delta\omega_D)(f_\alpha - f_\beta) e^{b^2} \operatorname{erfc}(b). \qquad (31)$$

Here n_o is the plasma density, f_α is the fraction of helium atoms in the upper laser state α, f_β is the fraction in the lower state β, $\Delta\omega_D$ is the Doppler broadening, and

$$b = \sqrt{\ln 2} \; {}^\alpha A_\beta / \Delta\omega_D . \qquad (32)$$

TABLE IV

ATOMIC STATE DEGENERACIES (g_α) AND ENERGIES (E_α)

Atomic State	α	g_α	E_α, eV
Cesium: 6 $^2S_{1/2}$	1	2	- 3.9
6 ^2P	2	-6	- 2.5
Helium: 1 ^1S_o	4	1	- 24.6
2 ^1S_o	5	1	- 4.0
2 $^1P_1$	6	3	- 3.4

TABLE V

PHOTON TRANSITION WAVELENGTHS AND PROBABILITIES

TRANSITION NUMBER, i	UPPER STATE NUMBER, α	LOWER STATE NUMBER, β	WAVELENGTH λ_i, Å	TRANSITION PROB $^\alpha A_\beta$, s^{-1}
1	6	4	584	1.8×10^9
2	6	5	20,851	2×10^6
3	2	1	8,857	3×10^6

Due to the plasma ion temperature (T_i ~ 50 eV), the Doppler broadening is determined by the thermal motion of the ions:

$$\Delta\omega_D = \nu_i \, \Delta v/c = 2 \, \pi\sqrt{3} \, T_i^{1/2}/\lambda_i m_i^{1/2} , \qquad (33)$$

where m_i is the mass of the ion. In writing Eq. (30), we have assumed that the photons travel in the z direction, as shown in Fig. 1. Since E_i^2 is proportional to the photon number density n_i, Eq. (30) may be written

$$\frac{\partial n_i}{\partial t} + c \frac{\partial n_i}{\partial z} = cg_i n_i - 2 \, ck_i n_i + n_\alpha \, {}^\alpha A_\beta F, \qquad (34)$$

where the last term is due to the spontaneous decay of the upper state α. Here F is the fraction of spontaneous photons which passes through the region of positive population inversion. Using $n_i = I_i/\hbar\nu_i c$, where I_i is the laser intensity and $\nu_i = 2\pi c/\lambda_i$, the gain term in Eq. (34) may be written $g_i I_i/\hbar\nu_i$. This is the stimulated emission term used in the helium atom rate equations (25) - (27). From Eq. (34), the equation for the laser intensity I_i is

$$\frac{\partial I_i}{\partial t} + c\,\frac{\partial I_i}{\partial z} = c\, g_i I_i - 2\, c k_i I_i + \hbar\, \nu_i c n_\alpha\, {}^{\alpha}A_{\beta} F. \quad (35)$$

The two photon rate equations represented by Eq. (35), where i = 1 and 2, and the eight particle rate equations (22) - (29) form a system of ten coupled equations which may be solved for the ten unknowns.

n_α, where $\alpha = 1,2,\ldots,8$

and

I_i, where $i = 1,2$.

VII. INITIAL AND BOUNDARY CONDITIONS

The cesium target is taken to be at rest, and the helium plasma approaches the target with velocity V_0 as shown in Fig. 1. At the leading edges of the cesium target and the plasma, the particle densities increase from zero to 10^{16} - 10^{19} cm^{-3}. We assume that the rise in cesium density is $[1-\exp(-x/x_{Cs})]$, where x_{Cs} is a scale length. The plasma density profile is of the same form with scale length x_{He}. The initial density profile of the target is shown in Fig. 3(a), where $n_\alpha^{initial}$ is the initial particle density in the interior of the target. Since the plasma is traveling toward the target, the rise in density of the plasma represents a boundary condition at the leading edge of the target. The rise in density of the plasma at x = 0 is shown in Fig. 3(b), where $t_{He} = X_{He}/V_0$.

We also simulate a ragged leading edge for the target as shown in Fig. 3(c), where the coordinate axes are oriented as in Fig. 1. Here ℓ is the number of lobes at the leading edge, and x_{rag} is the depth of the lobes. The target density is zero in the lobes, and then increases from the curve

$$x = x_{rag} \sin^2 (\ell\pi z/z_{max}), \quad (36)$$

where z_{max} is the laser length. We assume that the plasma leading edge is uniform along the z direction with no ragged edge effects.

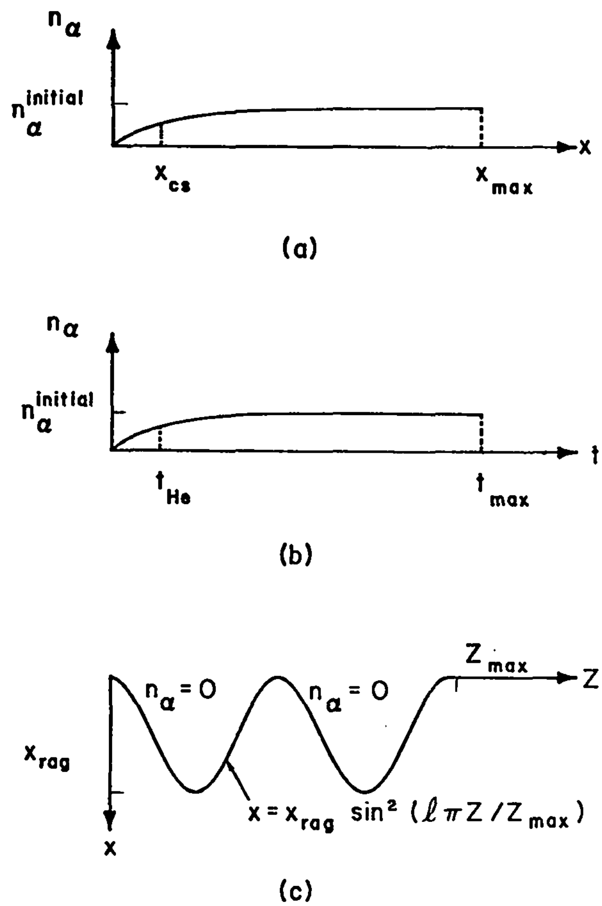


Fig. 3

VIII. RATE EQUATIONS IN DIMENSIONLESS FORM

We define the dimensionless particle density

$$D_\alpha = n_\alpha/n_o, \quad (37)$$

where n_o is the initial plasma density. The dimensionless penetration depth is

$$X = x\, {}^6A_4/V_0, \quad (38)$$

where ${}^6A_4$ is the transition probability of the 584-Å helium line given in Table V, and V_0 is the drift velocity. If $V_0 = 2.2 \times 10^7$ cm/s, which corresponds to drift energy $1/2\, m_i V_0^2 = 1$ keV, then the scale length $V_0/{}^6A_4$ is 0.12 mm. We define the dimensionless time

$$T = {}^6A_4\, t \quad (39)$$

and the dimensionless position along the laser axis

$$Z = z\ {}^6A_4/c \tag{40}$$

with scale length $c/{}^6A_4$ equal to 16.7 cm. From Eq. (16), the dimensionless charge exchange term is written

$$cx(\alpha,\beta)=D_7D_\beta\ {}_\alpha\rho_\beta[1-(D_3D\ /D_\alpha D_7)(g_\alpha/g_\beta)\exp(E_{\beta-\alpha}/T_i)], \tag{41}$$

where the charge exchange rate parameter is

$${}_\alpha\rho_\beta = n_o\ {}_\alpha R_\beta\ /{}^6A_4\ , \tag{42}$$

and ${}_\alpha R_\beta$ is given by Eq. (7). From Eq. (19), the dimensionless electron impact ionization term is

$$\mathrm{EION}(\alpha,\beta) = D_\alpha D_8\ {}_\alpha\sigma^\beta[1-(D_8D_\beta/D_\alpha)(h^2n_o^{2/3}/2\pi m_eT_e)^{3/2}$$

$$\times\ (g_\alpha/2g_o)\ \exp\ (E_{\alpha\text{-}\beta}/T_e)], \tag{43}$$

where the impact ionization rate parameter is

$${}_\alpha\sigma^\beta = n_o\ \ {}_\alpha S^\beta/{}^6A_4 \tag{44}$$

and ${}_\alpha S^\beta$ is given by Eq. (11). From Eq. (20), the electron impact excitation rate term is written

$$\mathrm{EXCITE}(\alpha,\beta)$$

$$= D_8D_\alpha\ {}_\alpha\sigma^\beta[1-D_\beta/D_\alpha)(g_\alpha/g_\beta)\exp(E_{\alpha-\beta}/T_e)]. \tag{45}$$

We define the dimensionless laser intensity

$$B_i = I_i/h\nu_1c\ n_o\ , \tag{46}$$

where $\nu_1 = 2\pi c/\lambda_1$ and λ_1 = 584 Å as given in Table V. The scale intensity $h\nu_1\ cn_o$ corresponds to photons with energy $h\nu_1$ and number density n_o and traveling with speed c. For plasma density $n_o = 10^{16}$ cm^{-3}, the scale intensity is 10^9 W/cm^2. The dimensionless gain is

$$G_i = c\ g_i/{}^6A_4\ , \tag{47}$$

where the scale gain is ${}^6A_4/c$ = 0.06 cm^{-1} and g_i is given by Eq. (31). The dimensionless stimulated emission term which appears in the particle rate equation is

$$\mathrm{STIM}(i) = \lambda_i\ G_i\ B_i/\lambda_1, \tag{48}$$

and the dimensionless spontaneous emission term is

$$\mathrm{SPONT}(i) = D_\alpha\ {}^\alpha A_\beta/{}^6A_4. \tag{49}$$

We also define the dimensionless photoabsorption term

$$\mathrm{PHOTO}(i) = 2k_i\ c\ \lambda_i/\lambda_i\ \ {}^6A_4.$$

Using the dimensionless quantities defined above, the particle rate equations (22)-(29) may be written in the dimensionless form

$$\frac{\partial D_1}{\partial T} = -\ {}_{\beta=4,5,6}\mathrm{CX}(1,\beta)-\mathrm{EION}(1,3)-\mathrm{EXCITE}(1,2)$$

$$+\ \mathrm{SPONT}(3)-\mathrm{PHOTO}(1)*B_1 \tag{50}$$

$$\frac{\partial D_2}{\partial T} = -\sum_{\beta=4,5,6}\mathrm{CX}(2,\beta)-\mathrm{EION}(2,3)+\mathrm{EXCITE}(1,2)$$

$$-\ \mathrm{SPONT}(3) \tag{51}$$

$$\frac{\partial D_3}{\partial T} = \sum_{\substack{\alpha=1,2\\ \beta=4,5,6}}\mathrm{CX}(\alpha,\beta) + \sum_{\alpha=1,2}\mathrm{EION}(\alpha,3)+\mathrm{PHOTO}(1)*B_1 \tag{52}$$

$$\frac{\partial D_4}{\partial T}+\frac{\partial D_4}{\partial X} = \sum_{\alpha=1,2}\mathrm{CX}(\alpha,4)-\mathrm{EION}(4,7)-\sum_{\beta=5,6}\mathrm{EXCITE}(4,\beta)$$

$$+\ \mathrm{SPONT}(1)\ +\ \mathrm{STIM}(1) \tag{53}$$

$$\frac{\partial D_5}{\partial T}+\frac{\partial D_5}{\partial X} = \sum_{\alpha=1,2}\mathrm{CX}(\alpha,5)-\mathrm{EION}(5,7)-\mathrm{EXCITE}(5,6)$$

$$+\ \mathrm{EXCITE}(4,5)+\mathrm{SPONT}(2)+\mathrm{STIM}(2) \tag{54}$$

$$\frac{\partial D_6}{\partial T}+\frac{\partial D_6}{\partial X} = \sum_{\alpha=1,2}\mathrm{CX}(\alpha,6)-\mathrm{EION}(6,7)+\sum_{\beta=4,5}\mathrm{EXCITE}(\beta,6)$$

$$-\sum_{i=1,2}[\mathrm{SPONT}(i)+\mathrm{STIM}(i)] \tag{55}$$

$$\frac{\partial D_7}{\partial T}+\frac{\partial D_7}{\partial X} = -\sum_{\substack{\alpha=1,2\\ \beta=4,5,6}}\mathrm{CX}(\alpha,\beta) + \sum_{\beta=4,5,6}\mathrm{EION}(\beta,7) \tag{56}$$

$$\frac{\partial D_8}{\partial T}+\frac{\partial D_8}{\partial X} = [-\sum_{\alpha=1,2}\mathrm{EION}(\alpha,3)-\sum_{\beta=4,5,6}\mathrm{EION}(\beta,7)$$

$$-\ \mathrm{EXCITE}(1,2)-\mathrm{EXCITE}(4,5)-\sum_{\alpha=4,5}\mathrm{EXCITE}(\alpha,6)]/\mathrm{NIMP}, \tag{57}$$

where NIMP ($\equiv T_e$/10 eV) is the number of events in which each hot electron participates. The photon rate equation (35) may be written in the dimensionless form

$$\frac{\partial B_i}{\partial T}+\frac{\partial B_i}{\partial Z} = G_iB_i\ -\mathrm{PHOTO}(i)*B_i\ +\ F\lambda_1\mathrm{SPONT}(i)/\lambda_i. \tag{58}$$

Equations (50) - (58) are the ten equations to be solved numerically.

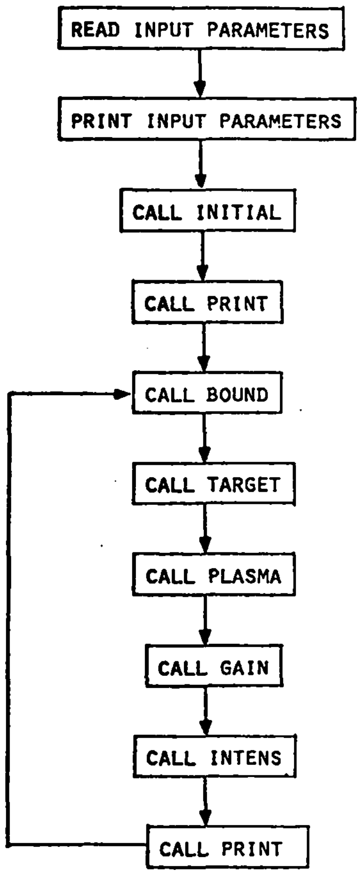


Fig. 4

IX. THE COMPUTER CODE

To permit use by others, the computer code is described. A simplified flow diagram for the main program is shown in Fig. 4.

The input parameters are punched on three data cards. The first data card contains values for the following FORTRAN variables:

DELT = time interval
DELX = penetration depth interval
DELZ = laser axis interval
TMAX = maximum value of time
XMAX = maximum penetration depth
ZMAX = laser length
NUMT = number of times calculated results are printed
NIMP = number of events in which each hot electron participates.

For example, if TMAX = 8 and NUMT = 4, the calculated results are printed at times T = 2, 4, 6, 8 for a total of four sets of output data.

Variables on the second data card determine the initial and boundary conditions:

DI(1) = initial density of the cesium ground state 6 $^2S_{1/2}$
DI(2) = initial density of the cesium 6 ^2P state
DI(3) = initial density of cesium ions
DI(4) = initial density of the helium ground state 1 ^1S_o
DI(5) = initial density of the helium 2^1S_1 state
DI(6) = initial density of the helium 2^1P_o state
DI(7) = initial density of helium ions
DI(8) = initial density of plasma electrons
XCS = cesium density scale length
XHE = plasma density scale length
XRAG = depth of lobes on the target edge
LOBE = number of lobes on the target edge.

These FORTRAN variables correspond to the initial and boundary condition variables shown in Fig. 3. For example, DI(1) = $n_1^{initial}/n_o$ and XRAG = $x_{rag}\,{}^6A_4/V_o$.

Values for the following FORTRAN variables are contained on the third data card:

TEMPE = plasma electron temperature in eV(T_e)
TEMPI = plasma ion temperature in eV(T_i)
DZERO = plasma density in cm^{-3} (n_o).

After reading and printing the input parameters contained on the three data cards, the main program calls SUBROUTINE INITIAL. This subroutine assigns the initial values to the eight particle densities D(NUM), where NUM = 1, 2,...,8. The initial values of the gains G(1) and G(2) and the laser intensities B(1) and B(2) are set equal to zero. Here I = 1 is the 584-Å line, and I = 2 is the 2 μ line.

The main program next calls SUBROUTINE PRINT. This subroutine prints the time T, as shown in Fig. 5, and then the position along the laser axis from Z = 0 to Z = ZMAX. The penetration depth X, from X = 0 to X = XMAX, is printed along the left edge of the paper. The particle densities D(NUM), gains G(1) and G(2), and laser intensities B(1) and B(2) are printed as functions of X and Z.

The plasma densities (NUM = 4 - 8) at X = 0 are assigned in SUBROUTINE BOUND. The main program then calls SUBROUTINE TARGET in which the cesium densities D(1), D(2) and D(3) are calculated using Eqs. (50) - (52). This subroutine calls on FUNCTION CX(N,M), FUNCTION EION(N,M), FUNCTION EXCITE (N, M), and FUNCTION SPONT (I) for the various rate equation terms. The charge exchange and electron impact excitation and ionization rate parameters are calculated in FUNCTION RHO(N,M) and FUNCTION SIG(N,M), according to Eqs. (42) and (44). The plasma particle densities D(NUM), where NUM = 4, 5, 6, 7, and 8 are calculated in SUBROUTINE PLASMA using Eqs. (53) - (57). Population inversions are found from the particle densities, and the gains for the 584-Å and 2 μ lines are calculated in SUBROUTINE GAIN using Eqs. (47) and (31). The intensities of the 584-Å and 2μ radiation are calculated in SUBROUTINE INTENS according to Eq. (58). The calculated values are printed as shown in Fig. 5. The time is advanced by DELT, and new values are calculated and printed until T = TMAX is reached. Depending on the choice of step sizes, the program requires several minutes of CPU time on the CDC 7600 computer.

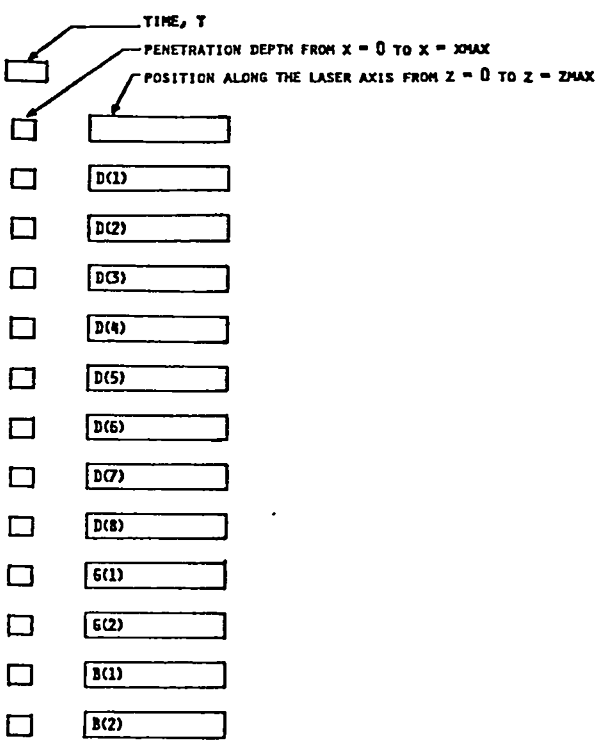


Fig. 5

X. NUMERICAL RESULTS

In this section we discuss the numerical solution of Equations (50)-(58) for the set of operating parameters[6] given in Table VI. The large cesium target density (10^{19} cm^{-3}) may be produced by vaporization from a flash-heated wire or plate. Since the target density is much greater than the plasma density ($n_{Cs} >> n_o$), the effects of electron impact ionization and excitation of the cesium atoms, reactions that compete with the charge exchange process which populates the upper laser level, are minimized.[7] That is, only a small fraction of the cesium ground state density is lost due to electron impact events.

TABLE VI
TYPICAL VALUES[6] FOR
THE OPERATING PARAMETERS

Parameter	Typical Value
Target Density	$n_{Cs} = 10^{19}$ cm^{-3}
Plasma Density	$n_o = 10^{16}$ cm^{-3}
Electron Temperature	T_e = 50 eV
Ion Temperature	T_i = 50 eV
Target Scale Length	x_{Cs} = 1.0 cm
Plasma Scale Length	x_{Cs} = 1.0 cm
Drift Velocity	$v_o = 2.2 \times 10^7$ cm/s

When the plasma intersects the target, charge exchange occurs in the interaction region and the upper laser state is populated. As the excited helium atoms drift deeper into the target, spontaneous and stimulated emission occur which fill the ground state of the helium atoms. Thus, a positive population inversion occurs near the edge of the target, and decreases with depth into the target. The depth at which the population inversion is a maximum is X = 0.8 (x = 0.1 mm) for the operating parameters listed in Table VI. The gain for the 584-Å line is in basic agreement with the analytical theory[2] for early time (T ≤ 4) when the stimulated emission and laser intensity are small. The gain is uniform along the laser axis, and the intensity increases nearly expotentially with Z as shown in Fig. 6. By time T = 8 (t = 4.5 ns), saturation effects begin to limit the laser intensity. The maximum intensity of 584-Å radiation is of order 10^6 W/cm^2.

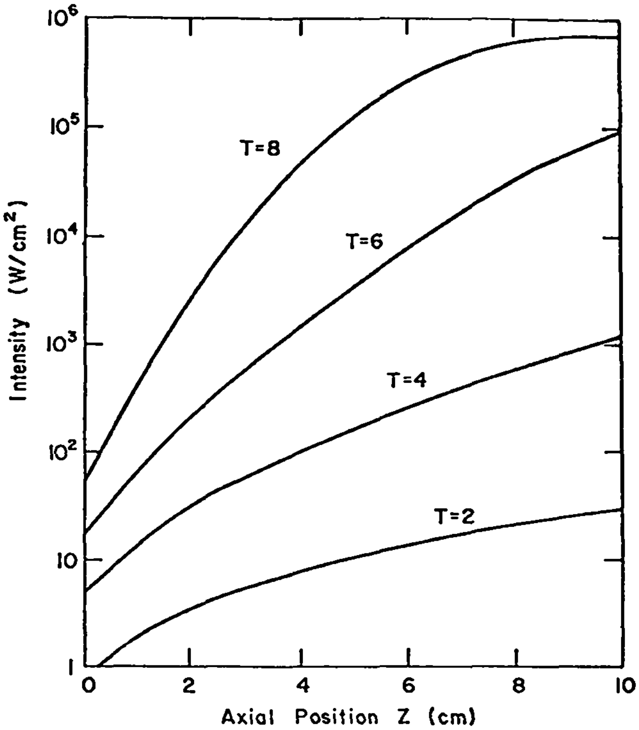


Fig. 6

The operating parameters have been varied about the typical values listed in Table VI with the following results:

(1) When the cesium density is increased to 10^{20} cm^{-3}, the laser intensity grows more rapidly as a function of time due to the increased charge exchange rate term [c.f. Eq. (14)]. Saturation effects occur at an earlier time (T = 6), and the maximum intensity remains of order 10^6 W/cm^2. Due to the increased target density, rapid charge exchange, and thus the maximum population inversion, occur closer to the target edge (x 0.04 mm). When the cesium density is reduced to the plasma density ($n_{Cs} = n_o = 10^{16}$ cm^{-3}), electron impact events destroy the cesium ground state population on a time-scale short compared to the spontaneous lifetime. That is, the population inversion is positive only for times T<1, and the radiation intensity is not significantly greater than the spontaneous background.

(2) When the drift velocity v_o is increased by a factor of 2, the intensity grows more rapidly as a function of time due to the increased charge exchange rate coefficient [c.f. Eq. (7)]. Since the faster moving helium atoms travel deeper into the target before undergoing spontaneous decay, the maximum population inversion occurs at a greater depth. The maximum laser output remains of order 10^6 W/cm^2. The charge-exchange cross section decreases when the drift energy is greater than $E_Q/2$ (= 5 keV from Table I). Although a computer run has not been made for this range of drift velocity, it seems reasonable to expect that the maximum laser intensity would be reduced.

(3) The maximum laser intensity is roughly proportional to the plasma density n_o. That is, the intensity is of order $10^7 W/cm^2$ for plasma density 10^{17} cm^{-3}.

(4) When the plasma temperatures are varied by 20 eV, the maximum laser intensity varies by less than an order of magnitude.

(5) A reduction in the density scale lengths x_{Cs} and x_{He} results in an increase in the intensity growth rate. This is because the cesium and helium densities build up more rapidly in the interaction region. The depth at which the population inversion is a maximum is closer to the target edge.

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